Perceived difficulty of a mathematical task: do in-service and preservice teachers have a common view?

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This qualitative study investigates how pre-service and in-service teachers perceive the difficulty of mathematical tasks and examines the factors that shape their evaluations. Using two tasks from the national large-scale assessments INVALSI, we aimed to identify whether perceived difficulties align with those of students and to explore any discrepancies between the two teacher groups. Our findings reveal significant differences between pre-service and in-service teachers, particularly in their handling of open-ended versus structured tasks. In-service teachers generally demonstrated alignment with students' evaluations for simpler tasks, while pre-service teachers exhibited emotions with respect to complex tasks. In our interpretation, these results highlight both the influence of teacher experience on perceived difficulty and the importance of addressing both cognitive and affective factors in teacher professional development.

Keywords: Mathematics education, perceived difficulty, pre-service teachers, in-service teachers, qualitative analysis.

Introduction

Difficulties in mathematics have been investigated in several researches, being one of the most open and studied topics in mathematics education. Difficulties can be classified into different categories depending on the focus: difficulties related to mathematics as a discipline or difficulties related to students in terms of deficits or their specific relationship with mathematics (Zan et al., 2006).

In our research, we focused on the difficulties of mathematical tasks, specifically on the *perceived difficulty* (PD) of in-service and pre-service teachers after solving a mathematical task. Literature distinguishes between difficulty *before* versus *after* a task has been attempted or solved (Eccles & Wigfield, 2020; Efklides & Touroutoglou, 2010); this is an important distinction in terms of perception of difficulty, yet in what follows we articulate a definition of PD that works in either cases. The difficulty of a mathematical task seems to depend on a set of factors, including the mathematical content (Thevenot & Oakhill, 2005; Radmehr & Drake, 2017), the formulation of the problem (Bolondi et al., 2018), text comprehension (Spagnolo et al., 2021), students' abilities or capabilities (Vicente et al., 2007), and affective factors (Zan et al., 2006). During the task-solving process, in fact, a student may encounter multiple difficulties that can depend on both the student's individual characteristics (such as abilities, knowledge, beliefs, and attitudes) and the specific characteristics of the task (such as the text or the mathematical content involved). These features of the mathematical task can influence both the student's and the teacher's perception of the task and, consequently, its PD. Thus, while closely connected, difficulty and PD are two different aspects (Spagnolo & Saccoletto, 2023). Furthermore, a student's PD does not necessarily coincide with the teacher's PD, as shown by Arzarello and Ferretti (2021). The mismatch between teachers' PD of a mathematical tasks and the actual success rates of those tasks opens up reflections related to research on teachers' beliefs about what students think (Philipp, 2007), as effective teaching and learning are believed to develop from the alignment between teachers' and students' beliefs (Kumaravadivelu, 1991). Our goal is to highlight how pre-service and in-service teachers perceive a task—whether it is easy or difficult—and what factors lead them to this evaluation, while also identifying potential mismatches. Specifically, we aim to investigate which view, between pre-service and in-service teachers' ones, is closer to the students' one.

Theoretical and context background

Currently, in Mathematics Education, there is no widely accepted definition of PD, despite its recognized importance as a factor influencing students' behavior when approaching and solving tasks (Eccles & Wigfield, 2020; Saccoletto & Spagnolo, 2022; Doz et al., 2023). In contrast, in metacognition research, the issue of subjective difficulty has been explored over the past 30 years under various terms (e.g., Eccles & Wigfield, 2020; Doz et al., 2023). At times, PD has been considered a form of self-efficacy, though this connection does not seem entirely appropriate (Eccles & Wigfield, 2020). However, related or overlapping concepts have been defined, such as the "feeling of difficulty" (FOD), which was elaborated by Efklides at various points between the late 20th century and 2011. FOD is described as a "metacognitive experience that monitors cognitive processing as it takes place" (Efklides & Touroutoglou, 2010, p.172) and is explicitly distinguished from PD due to its "experiential nature".

Although conceptually distinct, "feeling of difficulty" and PD are sometimes treated interchangeably (Nuutila et al., 2021). Acknowledging their differences, in this paper, we have drawn on some characteristics of both concepts and adapted them to the context of mathematics education research. From this perspective, the synthesis proposed by Doz et al. (2023) was especially helpful and aligned with our viewpoint. It suggests that the nature of the "feeling of task difficulty" is metacognitive, stemming from the monitoring of ongoing task processing, and awareness of this process influences self-regulation, effort, emotions, and strategy use.

The factors influencing PD of math tasks have mainly been studied in relation to students' perspectives (Saccoletto & Spagnolo, 2022). For our analysis, we began with the factors characterizing students' PD, as qualitatively identified by Saccoletto & Spagnolo (2022), and compared these to those reported by teachers in our data. According to this study, PD in mathematics can be classified into five categories: *Resolution Strategy*, *Capabilities and Experience*, *Emotions*, *Task Formulation*, and *Self Consideration*. Note that these categories are not mutually exclusive.

The category *Resolution Strategy* includes responses that explicitly mention the approach or method needed to solve the task, according to students, or highlight key elements like calculation or reasoning required to reach a solution. The category *Capabilities and Experience* encompasses answers that reflect students' views of their capabilities and abilities, as well as their prior experiences with similar tasks, which contribute to their familiarity with the problem. Answers that suggest a problem is easy because it resembles something previously encountered also fall into this category. Additionally, it includes responses related to students' self-perception, doubts about their solutions, and difficulties they faced, including the time spent on the problem. The category *Emotions* pertains to responses involving emotions. The category *Task Formulation* comprises responses that refer to the task's formulation, particularly its textual aspects. Finally, the category *Self Consideration* relates to students' personal reflections on their success in mathematics.

Research aim

Under these premises, our goal is to highlight how pre-service and in-service teachers perceive the difficulty of a mathematical task and to examine the factors that contribute to their evaluations. Furthermore, we aim to identify any discrepancies or differences between the perceptions of these two groups of teachers. A key focus of our investigation is to determine which of the two perspectives is more aligned with students' views of task difficulty, providing insight into how teacher experience influences their understanding of student challenges.

This study seeks to elucidate the multifaceted interplay between teacher experience and the PD of mathematical tasks. By employing a qualitative approach, the research not only investigates the cognitive dimensions of task evaluation but also delves into the affective responses that underpin teachers' judgments.

The research questions guiding this study are:

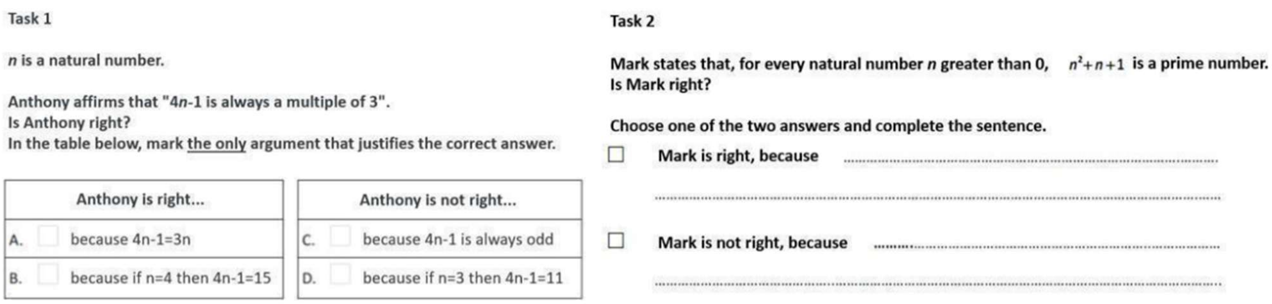
RQ1: How do pre-service and in-service teachers perceive the difficulty of a mathematical task, and what factors contribute to their evaluations?

RQ2: Are there notable discrepancies between the two groups of teachers?

RQ3: To what extent do their perceptions align with those of students?

Methods

We carried out a qualitative deductive study, involving 52 pre-service teachers and 49 in-service teachers of secondary school. All the teachers were from different Italian regions. Teachers filled in an online questionnaire through Google Forms, containing two tasks, some specific questions related to each task and three general questions. Each teacher worked on the questionnaire on their own, without knowing the other participants' answers. The questionnaire was the same used for students in Saccoletto & Spagnolo (2023), as we considered the categories determined by them for students, as an initial reference for teachers' results. We asked the teachers to solve the tasks (Figure 1) and, for each one, answer specific questions related to PD. Additionally, there were specific questions aimed to investigate attitudes and beliefs towards mathematics.


Task 1

n is a natural number.

Anthony affirms that "4*n*-1 is always a multiple of 3".
Is Anthony right?
In the table below, mark <u>the only</u> argument that justifies the correct answer.

Anthony is right...	Anthony is not right...
A. ☐ because 4n-1=3n	C. ☐ because 4n-1 is always odd
B. ☐ because if n=4 then 4n-1=15	D. ☐ because if n=3 then 4n-1=11

Task 2

Mark states that, for every natural number *n* greater than 0, n^2+n+1 is a prime number.
Is Mark right?

Choose one of the two answers and complete the sentence.

☐ **Mark is right, because** ....

☐ **Mark is not right, because** ....

Figure 1: Task 1 administered to Grade 08 Italian students by INVALSI in 2017 and task 2 administered to Grade 10 Italian students by INVALSI in 2014, www.gestinv.it

We choose two mathematics tasks from the INVALSI tests, as they are statistically validated and were also used in Spagnolo & Saccoletto (2023). In the Italian educational context, INVALSI tests—administered since 2008 in grades 2, 5, 8, 10, and 13 by the National Institute for the Evaluation of the Educational System—allow us to track some of the difficulties students experience over time.

These tests, designed to measure student competence in relation to the Italian curricular guidelines, offer a quantitatively validated tool that complements our qualitative analysis. Our focus was on argumentative questions within the Numbers area. The two tasks (Figure 1) share mathematical similarities, while also exhibiting some differences. Task 1 was a multiple-choice question requiring the identification of a correct argument, whereas Task 2 was an open-ended question that asked for the creation of an argument. Both tasks, however, were based on literal calculation and could be solved using the same strategy—proving the falsity of a statement by providing a counterexample. We remark that the two tasks have been given to students in grades 8 and 10 and they turn out to be difficult in the INVALSI tests, namely a few number of students gave the correct answer. In the study of Spagnolo and Saccoletto (2023) these tasks were administered to students at different grades: grade 8 students (at the end of the school year) and grade 9 students (at the beginning of the school year). The tasks were selected together with the class teachers.

The questions in the questionnaire for tasks 1 and 2 were identical and aimed at probing teachers' ideas, linking them—strictly in a qualitative way—to teachers' attitudes, beliefs, or specific features of the INVALSI tasks. The study adopted a deductive approach to data analysis, starting from the categories identified in Spagnolo and Saccoletto (2023) and working towards a broader understanding of overarching patterns and trends.

All analyses were carried out with Excel for the calculation of average values and for the graphical representation of the data, while the qualitative analysis of the teachers' evaluation of motivation was developed with the software MAXQDA 24 (v. 4.0), using as codes the macro-categories of factors influencing PD (Spagnolo & Saccoletto, 2023).

Results and discussion

Qualitative analysis of teachers' answers highlight important distinctions between how pre-service and in-service teachers perceive the difficulty of mathematical tasks, particularly when comparing Task 1 and Task 2.

Task 1, a multiple-choice question, was generally perceived as less challenging by both pre-service and in-service teachers, though the reasons behind their evaluations differed. In-service teachers, being more familiar with structured tasks, found Task 1 relatively straightforward. Pre-service teachers, while acknowledging the simplicity of the task, raised concerns regarding the clarity of the task formulation. Both groups perceived task 2, an open-ended question, as more difficult. On one hand, in-service teachers found the need for constructing a mathematical argument challenging, but their experience helped them cope with the complexity. Pre-service teachers, on the other hand, attributed much greater difficulty due to the open-ended nature of the task, reporting a reflection focused on the anxiety of answering an open-ended question. Below (Table 1), we present the distribution of responses in percentage terms for each category.

Student's PD does not seem to be related to the ability to answer the question correctly (Saccoletto & Spagnolo, 2022) and seem to refer much more often to the text, personal preparation, specific content, or aspects related to the production of explanation or argumentation to support their results. Also completely missing are aspects related to emotions, but much more present are self-considerations on how to improve one's performance.

We report the distribution of the categorized responses of students, pre-service and in-service teachers in the table below. In particular, for students we refer to the analysis in Spagnolo & Saccoletto (2023), so we report number and percentages for teachers, and only percentages for students.

Table 1: Distribution of the responses to Tasks 1 and 2 of students, pre-service and in-service teachers according to macro-categories identified in Saccoletto & Spagnolo (2023). The column with students' percentages is derived from Saccoletto & Spagnolo (2023).

Macro-categories	Students Task 1	Pre-service teachers Task 1	In-service teachers Task 1	Students Task 2	Pre-service teachers Task 2	In-service teachers Task 2
Resolution strategies	51,8%	40,38% (21 teachers)	59,18% (29 teachers)	33,6%	36,54% (19 teachers)	57,14% (28 teachers)
Capabilities and experience	39%	21,15% (11 teachers)	26,53% (13 teachers)	41,2%	30,77% (16 teachers)	46,94% (23 teachers)
Emotions	0%	36,54% (19 teachers)	6,12% (3 teachers)	2,9%	42,31% (22 teachers)	4,08% (2 teachers)
Task formulation	15,3%	28,85% (15 teachers)	18,37% (9 teachers)	5,3%	19,23% (10 teachers)	14,29% (7 teachers)
Self-considerations	47,8%	11,54% (6 teachers)	2,04% (1 teacher)	44,8%	5,77% (3 teachers)	4,08% (2 teachers)

Both for Tasks 1 and 2, pre-service teachers focused mostly (respectively 40,38% and 36,54% of teachers in each task) on *Resolution strategies*, identifying the correct strategy to solve the task, using logical elimination methods common in multiple-choice formats for Task 1, whereas for Task 2 emphasized the challenge of developing a strategy to construct an argument. For example, one teacher states that "if you often solve multiple choice questions, it becomes easier for you. That way you know it's important to start with the options and go by exclusion using logic". This answer was also categorised under *Capabilities and experience*. In-service teachers give also importance to experience and prior familiarity with similar tasks. 6% and 4% in-service teachers referred to the *Emotions* commenting Tasks 1 and 2 respectively, and 2% and 4% on *Self-Considerations* categories, commenting that "if I would have given more open-ended questions to my students during classes, they would have been less in trouble". In short, a few in-service teachers elicited emotions and self-considerations. Some teachers hypothesised problems in comprehension of the task (for both Task 1 and Task 2) connected with *Task formulation*. For example, one teacher stated, "For both tasks, I noticed that the wording could potentially cause some confusion, especially for students who struggle with calculations with letters. However, the instructions are clear enough for those with some experience in solving similar problems".

Pre-service teachers also focused on problem-solving strategies, in particular the answers of 21 pre-service teachers (for Task 1) and 19 pre-service teachers (for Task 2) were categorised below *Resolution strategies*. In both tasks, also *Capabilities and experience* and *Task Formulation* were dominant factors influencing PD of pre-service teachers. However, pre-service teachers frequently elicit emotional factors. Most of the answers under macro-category *Emotions* are related to negative emotions; for example, "Arguing something that has never been seen before in class [referring to '4n-1 is always a multiple of 3'] could generate a lot of anxiety in students", or "If you don't think of using a counterexample, proving a sentence in general can start panic". The emotional impact was most pronounced in Task 2 for pre-service teachers, highlighting the importance of preparing their future students to handle the cognitive and emotional demands of open-ended tasks.

In terms of alignment with student perspectives (for results of students see Spagnolo & Saccoletto, 2023), Task 1 was more easily aligned with students' evaluations, particularly by in-service teachers. This suggests that experienced teachers have a clearer understanding of how students approach and evaluate simpler, structured tasks. Task 2, with its open-ended format, revealed a greater mismatch, especially among pre-service teachers, who overestimated the difficulty, likely due to their limited experience in such tasks with students.

In direct response to our research questions, the results indicate that for RQ1, both pre-service and in-service teachers exhibit distinct perceptions of mathematical task difficulty, with pre-service teachers generally overestimating difficulty due to a greater emotional response and limited experience, whereas in-service teachers demonstrate a more balanced approach rooted in cognitive strategies and familiarity with task structures; in addressing RQ2, clear discrepancies are evident between the two groups, with pre-service teachers' evaluations being more influenced by affective factors and in-service teachers' judgments aligning more closely with analytical reasoning; and concerning RQ3, the alignment between teachers' and students' perceptions is notably stronger among in-service teachers, suggesting that practical experience enhances the ability to accurately estimate student difficulties.

Concluding remarks

The aim of the research is to compare how in-service and pre-service teachers perceive the difficulty of a task, and to see how much they differ from students' PD. PD has a strong affective connotation, as it entails beliefs, self-efficacy and emotions. The categories of resolution strategy, capabilities and experiences (which in our view are linked to beliefs), emotions, task formulation and self-consideration emerge in a previous study (Saccoletto & Spagnolo, 2023) and it has been observed that they might overlap, namely that a same teacher might consider more than one category when formulating their account for why they perceive a task as difficult or easy. Even if overlapping also in our study, these categories emerge with different intensity among groups of in-service and pre-service teachers. We have noticed, in fact, that resolution strategies, capabilities and experiences, and task formulation categories emerge in in-service teachers' accounts. This is also in line with literature on *difficulty* of mathematical tasks, as Vicente *et al.* (2007) stress the central role of students' abilities and capabilities, Bolondi *et al.* (2018) and Spagnolo *et al.* (2021) maintain that difficulty depend on the formulation of the problem and text comprehension, respectively, and Thevenot and Oakhill (2005) and Radmehr and Drake (2017) link difficulty to the mathematical content, which can be related to the category of resolution strategies.

We have also highlighted that Zan *et al.* (2006) argue that affective factors play a central role in the difficulty of a task, and pre-service teachers in our study focus on emotions quite significantly.

The students' perceptions are more in line with in-service teachers' ones, as if, on one side, as Kumaravadivelu (1991) posits it, there is a (positive, in our view) alignment between students' and teachers' beliefs, but on the other side it seems that emotions are elicited by those who are somehow further from everyday life in mathematical classrooms, namely pre-service teachers.

The relatively small sample size and qualitative nature of the study limit the generalizability of the findings. However, the consistent trends identified across both tasks provide a useful foundation for further exploration into how task type and teacher experience influence PD of mathematical tasks. Moreover, despite the differentiation of factors influencing PD, our definition appears to be adequate in capturing the perception of difficulty across different cases. The categories we identified, while with different distributions, help us categorize the responses of both groups of teachers (pre-service and in-service). They highlight the multifaceted nature of the construct, reinforcing the idea that PD encompasses both cognitive and affective dimensions, as well as task-related and individual-related aspects.

In conclusion, our results indicate clear differences in how pre-service and in-service teachers perceive the difficulty of tasks. Experience plays a key role in shaping these perceptions, and understanding these dynamics is crucial for improving teacher professional development and ensuring that teachers are better equipped to assess and address students PD face in solving mathematical tasks.

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